TEACH[R]

Using bendable and rigid manipulatives in primary mathematics

Is one more effective than the other in conceptualising 3D objects from their 2D nets?

Jacqui Scott
Teacher, North West Christian School, Penguin, Tasmania

Anton Selvaratnam
Senior Lecturer in Mathematics, Faculty of Education and Science, Avondale College of Higher Education, Cooranbong, NSW

Lynden Rogers
Senior Lecturer in Physics, Faculty of Education and Science, Avondale College of Higher Education, Cooranbong, NSW

Abstract

The usefulness of manipulatives in the primary maths classroom has been frequently asserted. The purpose of this study was to compare the effectiveness of two different types of manipulatives, *bendable* and *rigid*, as aids for the conceptualisation of 3D solids from 2D nets (fold-outs of solid geometrical shapes) within the NSW Stage 2 Mathematics Curriculum.

Contrary to initial expectations, the bendable nets, although more attractive to pupils, did not prove superior to the rigid variety. In fact, the most noticeable advances in conceptualisation followed teaching experiences using the rigid nets. Although this was a preliminary study and the sample sizes were too small to support solid conclusions, it is suggested that the data were sufficiently robust to warrant further investigation.

We suggest that the lower than expected results for the bendable nets may be explained, partially, by the reduced conceptual demands made by these more 'obvious' shapes. Correspondingly, the greater mental visualisation required when working with the rigid nets may have produced heightened student conceptualisation.

Introduction

In mathematics the term "manipulatives" is generally applied to any structured or unstructured materials and objects—which are physically handled by students—that allow them, actively and safely, to explore maths concepts and ideas. It has been recognised over several decades that the perceptive use of manipulatives enhances mathematics learning among primary and secondary students (cf. Yabsley, 1962; Dienes, 1964; Martinie and Stramel, 2004; Reys et al., 2007; Shaw, 2002) and some attention has also been given to their geometrical applications (Obara, 2009). Barger and McCoy (2009) have even argued the value of manipulatives for teaching geometry at tertiary level. However, little appears to have been done on the use of manipulatives in relating 2D nets to their corresponding 3D solids at the Stage 2 mathematics curriculum level. Further, although manipulatives may be constructed which allow differing degrees of 'manipulation' by the student, and which thus display different levels of correspondence to the concept under investigation, there have been no reports of the relative effectiveness of these different types of manipulatives. This study presents preliminary results from such an investigation.

Geometry is one of the oldest branches of mathematics. It has important connections to most other mathematical disciplines and much of life's experience. Despite its relevance, recent decades have seen geometry's substantial displacement by other topics in the mathematics classroom. These considerations suggest the importance of those geometry topics retained in the current primary curriculum and of instructional strategies which enhance their assimilation.

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The manipulatives

A rigid 2D net corresponding to any regular 3D solid, such as a prism, cube or pyramid, may be cut out of any flat medium. Similar 2D nets can be made which do *not* correspond to any 3D solid, due to the transposition of one or more sides. Some materials allow the construction of such nets with the additional capability of bending up into their 3D solid, thus providing a more obvious correspondence between the two forms. It should be noted here that care must be taken with terminology when discussing nets. For example, Ainge (1996, p. 346) defines a net merely in terms of the bendable variety, being a "plane diagram showing all faces of a 3D shape, which can be cut out and folded to construct the solid".

The medium chosen for this study was flute board—a safe, plastic sheet product available from office supply stores in a variety of bright colours. Cost, ease of handling and storage considerations suggested sizes for the 3D solids in the order of 3–6 cm side length. The bendable examples were made by systematically cutting away one side of the sheeting with a "V" cut using an angled picture-framing trimmer.

Two different sets of 2D nets were designed and constructed, each including examples which did correspond to 3D solids, and others which did not. Five solids were represented in these sets: Cube, rectangular prism, hexagonal prism, square-based pyramid and triangular-based pyramid. One set consisted of rigid nets and the other of bendable nets. Each set consisted of 15 different nets.

> *"The tests deliberately included some shapes not represented in the manipulative sets, to test depth of under-standing rather than prior knowledge and skills"*

Figure 1: Primary students reacted to the manipulatives with enthusiasm

Methodology

The following processes were completed before data collection commenced: Ethical clearance; consent from school administrators, parents and the class teacher; scheduling of class periods with the teacher and the preparation of resources and materials.

To begin, the Grade 4 class was split into two groups of 12 students with the assistance of the supervising class teacher. Originally it was intended that these two groups be approximately equal in ability but, as will be seen, the pre-test indicated that in the context of this study Group A was more able than Group B. However, this may have ultimately proved an advantage to the study, since it provided results for groups of different ability. A maximum of seven 50-minute time periods was allocated to the investigation by the class teacher. The study was consequently configured within these constraints; some time slots in the seven periods being available to the class teacher for regular maths.

Two worksheets were constructed: W1 (bn) corresponding to the bendable nets and W2 (rn) corresponding to the rigid nets. Both worksheets consisted of 15 questions, where each question related to a particular net. Students were allowed 90 seconds with each net during which to answer the appropriate worksheet question, after which the nets were rotated. As later explained, these worksheets were used in the periods following the familiarisation exercises.

Three 45-minute tests of identical format and structure were also constructed. These were designated T0 (pre), T1 (bn) and T2 (rn). Each had the same number and type of questions in each section. Although each included different selections of nets every attempt was made to produce three tests of similar difficulty. Appropriate to Grade 4, the 3D polyhedra included were:

- Prisms: Triangular, rectangular, pentagonal, square and hexagonal;
- Pyramids: Triangular, square, hexagonal, and pentagonal; and
- Cylinders and cones.

These tests included some shapes not represented in the manipulative sets with which pupils would have experience. This was done deliberately to test depth of understanding rather than prior knowledge and skills. The principal features of these tests were questions relating to whether or not a given 2D net accurately corresponded to any 3D solid, and if so, which one. These were paper tests, for which the pupils did not have access to the nets. The worksheets and tests were deployed according to a sequential schedule.

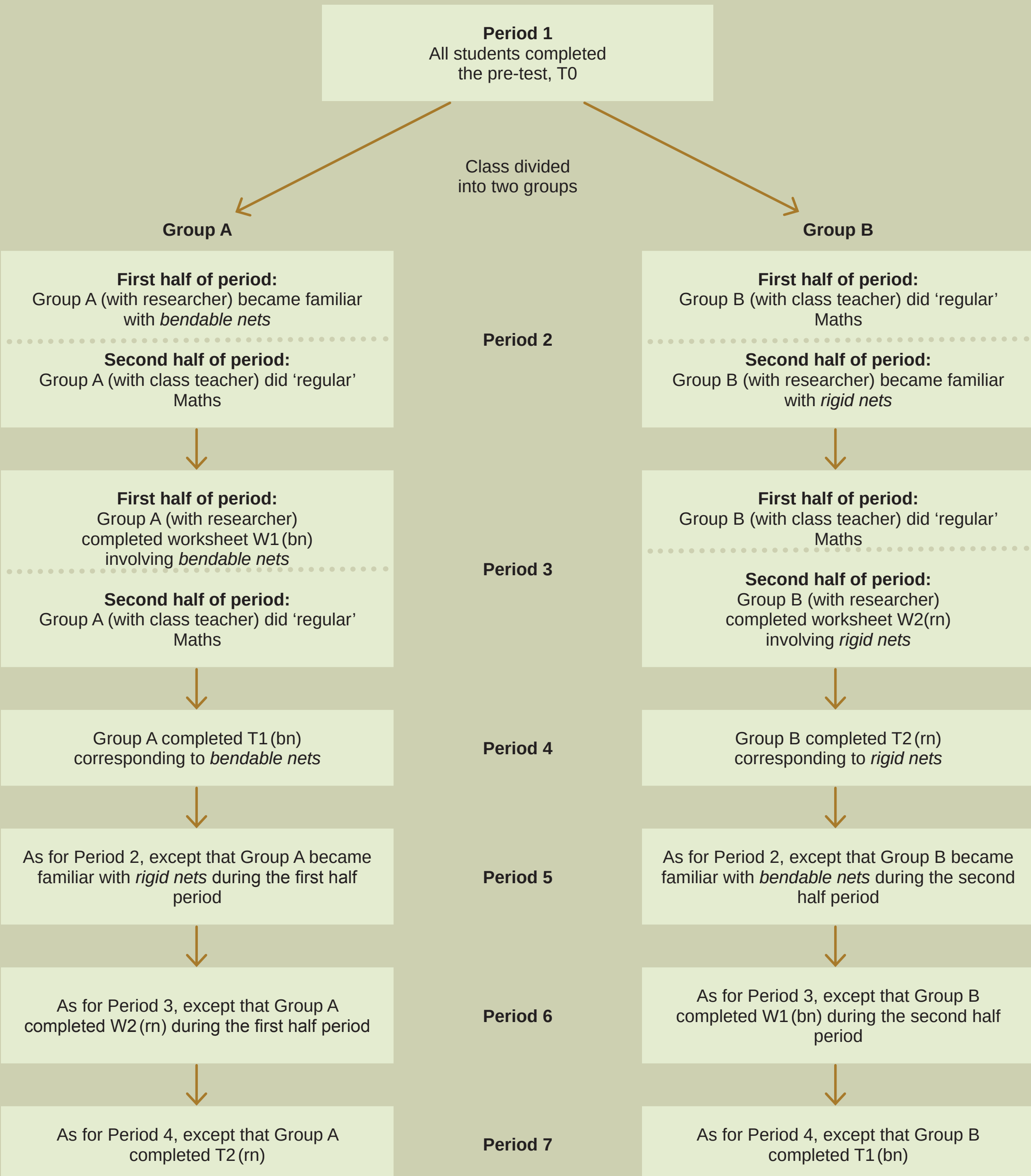
The *sequence* of the data collection process is illustrated by Figure 2.

Results

The following analysis only included the scores of those students who completed all three tests. This reduced the sample size down to nine students in each group. Table 1 shows the mean test percentages obtained and the *t*-test data emerging from comparisons of the (means of dependent samples) results from Tests 1 and 2 to those from Test 0. Right-tailed *t*-testing was performed since

> *The analysis only included the scores of students who completed all three tests*

Figure 2: Stages of the data collection process: An overview



> *The largest increase in test mean followed the teaching experience involving the rigid nets*

this study was searching for test improvement only. Also shown are the associated "p" values and the Cohen's effect size values, "d". Table 2 shows the mean percentages for the worksheets W1 (bn) and W2 (rn).

As may be seen from Table 1, the overall outcome of both "net" learning opportunities was an improvement in mean test scores of about 14%, a change that is significant and 99% certain for Group A and 94% certain for Group B (just below the convention for significance, this being greater than 95%). For each, the associated Cohen's effect size measure "d" was greater than 0.5, indicating a large effect. Further, the largest increase in test mean followed the teaching experience involving the rigid nets. This was true for both groups and to a very similar degree. For Group A (the more able group) there was a 3.2% mean increase in test mean following experience with the bendable nets and a further, much larger improvement of 10.9% after the rigid nets. Group B showed a 9.6% increase in test mean following experience with the rigid nets and a further increase of only 4.0% when exposed to the bendable nets.

The right-tailed *t*-test analysis strengthened this observation. For our small sample size the critical *t*-value corresponding to a 98% significance level was 2.90. As may be seen, the T2 (rn) / T0 (pre)

Table 1: Test results for both groups

Group A		
Result for T0 (pre) (pre-test)	**Result for T1 (bn)** (following learning experience with bendable nets)	**Result for T2 (rn)** (following learning experience with rigid nets)
65.7%	68.9%	79.8%
←	$t_{21} = 0.46$ →	
←	$t_{31} = 2.95$	→

Group B		
Result for T0 (pre) (pre-test)	**Result for T2 (rn)** (following learning experience with rigid nets)	**Result for T1 (bn)** (following learning experience with bendable nets)
55.8%	65.4%	69.4%
←	$t_{31} = 2.89$ →	
←	$t_{21} = 1.72$	→

Table 2: Worksheet results for both groups

Group A	
Result for W1 (bn) (following learning experience with bendable nets)	**Result for W2 (rn)** (following learning experience with rigid nets)
75.3%	71.5%

Group B	
Result for W2 (rn) (following learning experience with rigid nets)	**Result for W1 (bn)** (following learning experience with bendable nets)
55.7%	77.9%

comparisons for both groups gave *t*-values very close to, or exceeding, this critical value, with p-value < 0.01. This indicated that the improvement following instruction with rigid nets was very unlikely to be a chance result. Each associated Cohen's "d" effect size measure was approximately 0.7; considerably greater than 0.4, the level suggested by Hattie (2012) as indicating large effects. However, the T1 (bn) / T0 (pre) comparisons gave results which were less significant for both groups. Interestingly, the different ordering of the tests appeared not to have greatly affected these results. These data suggest that the teaching experience using rigid nets produced a statistically significant improvement in test score, whereas this can not be said of that involving the bendable variety.

Clearly, one possible explanation of these disparities is that T2 (rn), which followed the learning experience involving rigid nets for both groups, was easier than T1 (bn). The reason why T1 (bn) and T2 (rn) had each been associated with just one type of net was to facilitate comparisons between the two groups. However, different levels of test difficulty would compromise these comparisons. In order to check this possibility, T1 (bn) and T2 (rn) were retrospectively submitted to four academic peers with mathematical experience, all of whom were asked to complete them and compare their difficulty. All four rated the tests as very close, there being no predominant judgement of one being more difficult than the other. This implies that the results obtained were not an artefact of uneven test difficulty.

Another objection which might be raised is that since the tests themselves feature "rigid" nets, i.e. ones drawn on paper, it is somewhat predictable that students will perform best after completing learning experiences with rigid nets. There may be some validity to this point and further work could be done on devising a more objective means of evaluation.

Conclusions

For Group A there was clearly a much bigger improvement in conceptualisation following class experience with the rigid nets than with the bendable variety. This was contrary to our initial expectations and gave rise to the suspicion that some of this improvement may be simply attributed to accumulating experience, since this group experienced the rigid nets last. However, Group B showed a similar pattern with a reversed order of contact, suggesting that the experience factor was not significant.

Students using the bendable nets could identify their 3D shape and whether or not they 'worked' by actually bending and seeing. It is then no surprise that the worksheet results showed higher levels of performance when using the bendable nets than for the rigid variety. This was particularly true of the less able Group B, which might be expected. It is important to note here that providing such an opportunity to experience success is of primary motivational importance to teaching students of lower ability.

When learning with rigid nets, students had to identify the corresponding 3D shape and decide whether or not they 'worked' from their flat configuration alone. They were therefore forced to manipulate the shapes in their minds rather than with their hands, focusing on mental rather than physical processing. Thus, although not performing as well for the worksheet, it appears that the rigid nets required and developed superior abstract thinking in identifying 3D shapes from flat nets, giving rise to more significant performance improvements in the written test.

There are additional ways in which this study could be developed. Larger sample sizes would allow a better assessment of the very tentative findings of this study. It would also be of interest to test these conclusions using a different type of instrument, as noted above. A gender comparison of the conceptualisation of 3D shapes might also prove instructive.

As evident from Figure 1, pupils thoroughly enjoyed working with the manipulatives, the bendable variety being definitely the more popular. This supports the idea that the use of a range of tactile experiences in the classroom not only diversifies 'assimilation' pathways, but makes learning more enjoyable. TEACH

> *"The use of a range of tactile experiences in the classroom not only diversifies 'assimilation' pathways, but makes learning more enjoyable"*

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